

Calculus 1  
Exam 1  
17 February 2020

Name: Key

• **Instructions:**

- Be sure to read each problem's directions.
- Write clearly during the exam and fully erase or mark out anything you do not want graded.
- You may use your calculator for any calculation or derivation unless otherwise stated so long as you show your work leading up to that point and then state that that is what you are doing.
- **You must show all your work to receive full credit unless otherwise stated.**
- *Don't cheat. See below.*

• **Prohibited:**

- Class Notes, Handouts
- Homework Notebooks
- Study Guides and Materials
- Previous Exams or Exam Solutions
- The Book
- Any Electronics (including phones) besides an approved TI Calculator
- Other People's Exams

Page:	2	3	4	5	6	7	8	Total
Points:	25	20	30	30	15	15	15	150
Score:								

**Multiple Choice**

1. For the following multiple choice questions, you must circle both the word True or False as well as the letter corresponding to the correct answer. You do **not** have to show any work for the multiple choice parts.

(a) (5 points) The domain of the inverse cosine function is  $[0, \pi]$ .

A. True  B. False

(b) (5 points)

$$\lim_{x \rightarrow 4^+} \sqrt{x^2 - 16} = 0$$

A. True  B. False

(c) (5 points) The greatest integer or floor function,  $f(x) = \lfloor x \rfloor$  is discontinuous at all integer values of  $x$ .

A. True  B. False

(d) (5 points) If the product function  $h(t) = f(t) \cdot g(t)$  is continuous at  $x = c$ , then both  $f(t)$  and  $g(t)$  must be continuous at  $x = c$ .

A. True  B. False

$$f(t) = \lfloor t \rfloor, g(t) = 0.$$

(e) (5 points) This question is a scale to ensure that you're actually reading these True/False questions. Draw a star next to the word True for full credit.

A. True  B. False



Points earned: \_\_\_\_\_ out of a possible 25 points

**Computation**

2. Compute the following limits by hand (No Calculators). If a limit does not exist, explain why using full sentences.

(a) (10 points)

$$\lim_{z \rightarrow 2} 5z^2 - z$$

$$\lim_{z \rightarrow 2} 5z^2 - z = 5(2)^2 - 2 = 18$$

(b) (10 points)

$$\lim_{x \rightarrow 12} \frac{x - 12}{x^2 - 144}$$

$$\begin{aligned} \lim_{x \rightarrow 12} \frac{x - 12}{(x - 12)(x + 12)} &= \lim_{x \rightarrow 12} \frac{1}{x + 12} \\ &= \frac{1}{24} \end{aligned}$$

Points earned: \_\_\_\_\_ out of a possible 20 points

(c) (10 points)

$$\lim_{x \rightarrow 0^+} 1 - \ln x$$

$$1 - \lim_{x \rightarrow 0^+} \ln x = \infty$$

(d) (10 points)

$$\lim_{t \rightarrow 6} \sqrt{t^2 - 36}$$

DNE (fail left limit)

(e) (10 points)

$$\lim_{y \rightarrow \frac{\pi}{2}^+} \csc y$$

$$\csc y = \frac{1}{\sin y}$$

$$\lim_{y \rightarrow \frac{\pi}{2}} \sin y = \sin \frac{\pi}{2} = 1$$

$$\lim_{y \rightarrow \frac{\pi}{2}^+} \csc y = \frac{1}{1} = 1$$

Points earned: \_\_\_\_\_ out of a possible 30 points

3. Consider the below table of a function  $f(t)$ .

$t$	5.9	5.99	5.999	5.9999	6.0001	6.001	6.01	6.1
$f(t)$	63.4	63.2004	63.20001	63.200001	63.19999	63.1999	63.199	63.1

(a) (15 points) What is the average rate of change of  $f$  over  $[5.99, 5.9999]$ ? Write your final answer in decimal notation with no fractions.

$$\frac{63.200001 - 63.2004}{5.9999 - 5.99} = -0.04\overline{03}$$

(b) (15 points) What is the limit of  $f(t)$  as  $t$  approaches 6? You do not need to show any calculations for this part but you must justify your answer using full sentences.

63.2

Left & Right limits converge numerically to this value.

4. (15 points) Determine all asymptotes of the following rational function by hand. You may use a calculator to check your answers but you must show all work to determine the asymptotes without aids.

$$j(x) = \frac{6x^4 - 59x^3 + 36x^2 + 521x - 840}{2x^3 + x^2 - 15x} = \frac{(x-8)(x+3)(2x-5)(3x-7)}{x(x+3)(2x-5)}$$

$$j(x) = \frac{(x-8)(3x-7)}{x} = \frac{3x^2 - 31x + 56}{x}$$

Hole:  $\boxed{x = -3, 5/2}$  (cancellations)

V.A.:  $\boxed{x = 0}$

H.A.:  $\boxed{\text{None}}$  (deg(Num) > deg(Den))

O.A.: deg(Num) = deg(Den) + 1

$$\begin{array}{r} 3x - 31 \\ x \overline{) 3x^2 - 31x + 56} \\ \underline{-(3x^2)} \phantom{+ 56} \\ -31x + 56 \end{array}$$

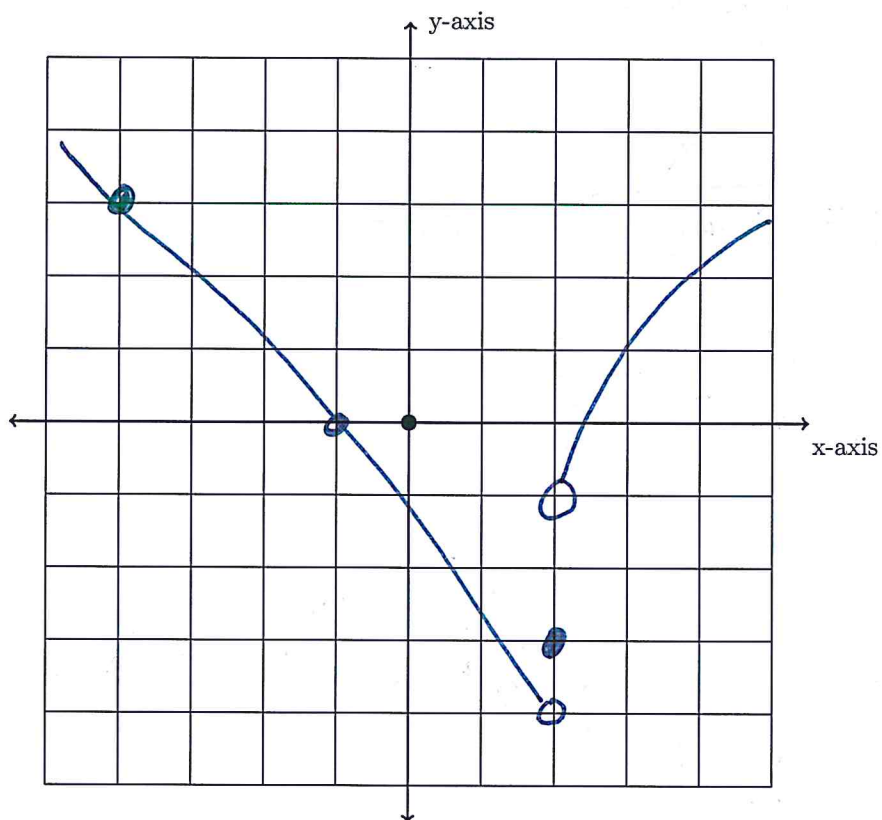
(could also use original rational function)

$$\boxed{y = 3x - 31}$$

**Concepts**

5. (15 points) Sketch the graph of a function  $k(x)$  with domain  $[-5, 5]$  with the following properties:

- $k(x)$  is continuous everywhere on  $[-5, 5]$  except at  $x = 2$
- $k(-4) = 3$
- $k(2) = -3$
- $\lim_{x \rightarrow -1} k(x) = 0$
- $\lim_{x \rightarrow 2^+} k(x) = -1$
- $\lim_{x \rightarrow 2^-} k(x) = -4$ .



Points earned: \_\_\_\_\_ out of a possible 15 points

6. (15 points) Determine a continuous extension for the below function by hand. You may use a calculator to check your answers but you must show all work to determine any possible continuous extension without aids.

$$f(x) = \frac{x^2 - 6x}{x^2 - 36}$$

$$f(x) = \frac{x(x-6)}{(x-6)(x+6)}$$

$$f(x) = \frac{x}{x+6} ; x \neq 6$$

$$\lim_{x \rightarrow 6} f(x) = \frac{6}{12} = \frac{1}{2}$$

$$F(x) = \begin{cases} \frac{x^2 - 6x}{x^2 - 36} & ; x \neq 6 \\ \frac{1}{2} & ; x = 6 \end{cases}$$

Points earned: \_\_\_\_\_ out of a possible 15 points